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XV.

AN EXAMINATION OF THE STANDARDS OF LENGTH CONSTRUCTED BY THE SOCIÉTÉ GÉNEVOISE.

By Professor W. A. Rogers.

Communicated December 10th, 1884.

Several physical laboratories in this country have recently received from the Société Génevoise instruments of precision of various kinds, which appear to have decided merit, both in regard to design and workmanship. The Society has among other things undertaken the construction of standards of length, and of a cathetometer, which is designed to take a high rank as an instrument of precision. Through the kindness of Professor Wright of Yale College, the writer has been permitted the opportunity of a somewhat extended study of one of the standard meters of the Society. Through the courtesy of J. W. Queen & Co. of Philadelphia, the opportunity was at the same time offered of an examination of three other standard meters, and of the meter graduated upon the upright bar of a cathetometer.

On account of the somewhat extended introduction of these standards, it has seemed to the writer worth while to place upon record the results of this examination.

EXAMINATION OF METERS.

The meter belonging to Professor Wright is designated W. The meter of similar form and dimensions received from J. W. Queen & Co. is designated Q_1. A second meter, in which the graduations are nearly along the centre of gravity of a cross section of the bar, is designated Q_2. The graduations upon W, Q_1, and Q_2 are upon silver inlaid in the brass, which is the material of the bars. A third meter, designated Q_3, has the graduations upon the brass. The meter of the cathetometer is designated Q_4.

COMPARISON OF METER W WITH BRONZE STANDARD METER $R_2{}^{a^2}$ AND STEEL STANDARD METER R_3,

in which, designating the Mètre des Archives by A_0,

$$R_2{}^{a^2} - 1.6\,\mu = A_0$$
$$R_3 \; - 1.2\,\mu = A_0$$

The coefficient of expansion of R_2 for 1° C. in 1 metre is assumed to be $17.17\,\mu$.* The coefficient of R_3 for 1° C. is assumed to be $10.28\,\mu$. At 0° C. these relations become, with these coefficients,

$$R_2{}^{a^2} + 284.6\,\mu = A_0$$
$$R_3 \; + 170.2\,\mu = A_0$$

The values $R_2 - W$ and $R_3 - W$ are given in divisions of the micrometer of the microscope employed, in which

$$1 \text{ div.} = 0.503\,\mu.$$

The thermometer employed is No. 8612 Baudin, and the readings τ have been reduced to the Yale standard.

The observations extend from November 15 to December 3, 1884.

EQUATIONS OF CONDITION BETWEEN R_2 AND W.

No. Obs.	$R_2 - W$		$(16°.67 - \tau)$	$R_2 - W$ at 16°.67	Residuals.	
3	—490.6 div.	$= a$	$+19.41\,b$	—526.1 div.	—3.2 div.	$= -1.6\,\mu$
3	—496.9	$= a$	$+14.79\,b$	—524.0	—1.1	$= -0.6$
3	—499.2	$= a$	$+13\;44\,b$	—523.8	—0.9	$= -0.5$
6	—501.0	$= a$	$+10.94\,b$	—521.0	+1.9	$= +1.0$
2	—508.4	$= a$	$+\;8.07\,b$	—523.2	—0.3	$= -0.2$
4	—512.2	$= a$	$+\;5.31\,b$	—521.7	+1.2	$= +0.6$
5	—513.3	$= a$	$+\;1.68\,b$	—516.4	+6.5	$= +3.3$
4	—552.5	$= a$	$-14.06\,b$	—526.8	—3.9	$= -2.0$
			Mean	—522.9		

Normal Equations.

$$-\;4073.9 = 8\,a + 59.58\,b$$
$$-28977.3 = 59.58\,a + 1189.62\,b$$

Whence

$$a = -522.9 \text{ div.} = -263.0\,\mu$$
$$b = +\;1.83 \text{ div.} = +\;0.92\,\mu$$

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* Proceedings of the American Academy, Vol. XVIII. p. 341.

EQUATIONS BETWEEN R_3 AND W.

No. Obs.	$R_3 - W$		$R_3 - W$ at 16°.67	Residuals.	
3	—220.4 div.	$= a + 19.41\,b$	—522.0 div.	+2.1 div.	$= +1.1\,\mu$
4	—301.4	$= a + 14.79\,b$	—531.2	—7.1	$= -3.6$
3	—313.7	$= a + 13.44\,b$	—522.6	+1.5	$= +0.8$
6	—351.2	$= a + 10.94\,b$	—521.2	+2.9	$= +1.5$
2	—397.1	$= a + 8.07\,b$	—522.5	+1.6	$= +0.4$
4	—439.1	$= a + 5.31\,b$	—521.6	+2.5	$= +1.3$
5	—501.5	$= a + 1.68\,b$	—527.6	—3.5	$= -1.8$
4	—742.3	$= a - 14.06\,b$	—523.8	+0.3	$= +0.2$
		Mean	—524.1		

Normal Equations.

$$-3266.7 = 8\,a + 59.58\,b$$
$$-12736.4 = 59.58\,a + 1189.62\,b$$

Whence

$$a = -524.1 \text{ div.} = 263.6\ \mu$$
$$b = +15.54 \text{ div.} = 7.82\,\mu$$

It appears from these observations, that the coefficient of W for 1° C. from comparison with R_2 is

$$17.17\,\mu + 0.92\,\mu = 18.09\,\mu;$$

and that the coefficient of W for 1° C. from comparison with R_3 is

$$10.28\,\mu + 7.82\,\mu = 18.10\,\mu.$$

On account of the large deviation from the value communicated by the Society, viz. 19.155 μ, a series of comparisons was instituted beween W and a steel end-meter S immersed in melting ice, according to the method described in the Proceedings of the American Academy, Vol. XVIII. p. 341.

EQUATIONS OF CONDITION BETWEEN W AND S IN MELTING ICE.

No. Obs.	$S - W$	$(16°.67 - \tau)$	$S - W$ at 16°.67	Residuals.	
5	+ 30.0 div.	$= a + 17.75\,b$	—619.1 div.	—0.8 div.	$= -0.4\,\mu$
5	— 7.5	$= a + 16.66\,b$	—616.8	+1.5	$= +0.8$
10	— 177.0	$= a + 12.24\,b$	—624.6	—6.3	$= -3.2$
9	— 198.3	$= a + 11.58\,b$	—621.8	—3.5	$= -1.8$
5	— 260.9	$= a + 9.58\,b$	—611.2	+7.1	$= +3.6$
6	— 327.4	$= a + 7.94\,b$	—617.8	+0.5	$= +0.3$
4	— 369.8	$= a + 6.73\,b$	—615.9	+2.4	$= +1.2$
10	— 571.5	$= a + 1.28\,b$	—618.3	+0.0	$= +0.0$
18	—1124.7	$= a - 13.83\,b$	—618.9	—0.6	$= -0.3$
		Mean	—618.3		

Normal Equations.

$$-3007.1 = 9a + 69.93\,b$$
$$+3180.1 = 69.93\,a + 1269.56\,b$$

Whence

$$b = 36.57 = 18.39\,\mu$$

Combining this value with the values 18.09 μ derived from R_2, and 18.10 μ derived from R_3, we have:

Coefficient of expansion of bar W in one meter = 18.19 μ for each degree Centigrade.

For the relation between W and A_0 at 16°.67 C. = 62°.0 Fahr., we have:

$$W - 263.0\,\mu = R_2 = A_0 + 1.6\,\mu \qquad W - 263.6\,\mu = R_3 = A_0 + 1.2\,\mu$$
$$W - 264.6\,\mu = A_0 \qquad W - 264.8\,\mu = A_0$$

And finally, $$W - 264.7\,\mu = A_0$$

Reducing to 0° C., with the coefficient 18.19 μ, we have at 0°

$$W + 38.5\,\mu = A_0$$

A slightly different but probably more accurate value of this relation will be obtained by selecting only those comparisons which were made near 0°. From these data we have:

No. Obs.	8612 °	$R_2 - W$	$R_2 - W$ at 0° C.	No. Obs.	8612 °	$R_3 - W$	$R_3 - W$ at 0° C.
3	−2.74	−490.6 div.	−249.6 μ	3	−2.74	−220.4 div.	−132.6 μ
3	+1.88	−496.9	−247.9	4	+1.88	−301.4	−136.9
3	+3.43	−499.2	−247.6	3	+3.00	−313.7	−132.4
6	+5.73	−501.0	−246.2	6	+5.73	−351.2	−132.2
		Mean	−247.8				−133.5

Whence $W + 36.1\,\mu = A_0$ $\qquad$ $W + 36.0\,\mu = A_0$

COMPARISON OF STANDARD Q_1 WITH STANDARDS R_2 AND R_3.

EQUATIONS OF CONDITION BETWEEN R_2 AND Q_1.

No. Obs.	$R_2 - Q_1$		$R_2 - Q_1$ at 16°.67	Residuals.	
6	−580.2 div.	$= a + 16.02\,b$	−604.9 div.	+1.0 div.	= +0.5 μ
4	−586.9	$= a + 12.54\,b$	−606.2	−0.3	= −0.2
2	−597.1	$= a + 6.59\,b$	−607.2	−1.3	= −0.6
4	−627.1	$= a - 14.06\,b$	−605.4	+0.5	= +0.3
		Mean	−605.9		

Normal Equations.

$$-2391.30 = 4a + 21.09b$$
$$-11772.39 = 21.09a + 655.00b$$

Hence

$$a = -605.9 = 304.8\ \mu$$
$$b = +\ 1.54 = 0.78\mu$$

$$Q_1 - 304.8\mu = R_2 = A_0 + 1.6\mu$$
$$Q_1 - 306.4\mu = A_0$$

$$\text{Coefficient} = 17.17\mu + 0.78\mu = 17.95\mu$$

EQUATIONS OF CONDITION BETWEEN Q_1 AND R_3.

No. Obs.	$R_3 - Q_1$		$R_3 - Q_1$ at 16°.67	Residuals.		
6	—364.2 div.	$= a + 16.02b$	—603.4 div.	+3.3 div.	=	+1.7 μ
4	—424.2	$= a + 12.54b$	—611.4	—4.7	=	—2.4
2	—506.9	$= a + 6.59b$	—605.4	+1.3	=	+0.7
4	—816.5	$= a - 14.06b$	—606.6	+0.1	=	+0.1

Normal Equations.

$$-2111.8 = 4a + 21.09b$$
$$-3014.4 = 21.09a + 655.00b$$

Hence

$$a = -606.7 = -305.2\ \mu$$
$$b = +\ 14.93 = +\ 7.51\mu$$

$$Q_1 - 305.2\mu = R_3 = A_0 + 1.2\mu$$
$$Q_1 - 306.4\mu = A_0$$

$$\text{Coefficient} = 10.28\mu + 7.51\mu = 17.79\mu$$

COMPARISON BETWEEN STANDARDS Q_2, R_2, AND R_3.

EQUATIONS OF CONDITION BETWEEN R_2 AND Q_2.

No. Obs.	$R_2 - Q_2$		$R_2 - Q_2$ at 16°.67	Residuals		
6	—530.8 div.	$= a + 16.03b$	—574.5 div.	—3.8 div.	=	—1.9 μ
4	—533.9	$= a + 12.96b$	—569.2	+1.5	=	+0.8
3	—539.1	$= a + 10.40b$	—567.5	+3.2	=	+1.6
4	—609.9	$= a - 14.06b$	—571.5	—0.8	=	—0.4

Normal Equations.

$$-2213.7 = 4a + 25.33b$$
$$-12459.4 = 25.33a + 730.76b$$

Hence

$$a = -570.7 = -287.1\ \mu$$
$$b = +\ 2.73 = +\ 1.45\mu$$

$$Q_2 - 287.1\mu = R_2 = A_0 + 1.6\mu$$
$$Q_2 - 288.7\mu = A_0$$

$$\text{Coefficient} = 17.17\mu + 1.45\mu = 18.62\mu$$

Equations of Condition between Q_2 and R_3.

No. Obs.	$R_3 - Q_2$		$R_3 - Q_2$ at 16°.67	Residuals.		
6	—324.4 div.	$= a + 16.03\,b$	—579.4 div.	—5.5 div.	=	—2.8 μ
4	—361.6	$= a + 12.96\,b$	—567.7	+ 6.2	=	+3.1
3	—408.3	$= a + 10.40\,b$	—573.8	+0.1	=	+0.1
4	-798.3	$= a - 14.06\,b$	—574.6	—0.7	=	—0.4

Normal Equations.

$$-1892.6 = 4\,a + 25.33\,b$$
$$-2908.6 = 25.33\,a + 730.76\,b$$

Hence

$$a = -573.8 \text{ div.} = 288.6\ \mu$$
$$b = +\ 15.91 \text{ div.} = 8.00\,\mu$$

$$Q_2 - 288.6\,\mu = R_3 = A_0 + 1.9\,\mu$$
$$Q_2 - 289.8\,\mu = A_0$$

Coefficient $= 10.28\,\mu + 8.00\,\mu = 18.28\,\mu$

Comparison of Q_3 with R_2.

	8612 °	$R_2 - Q_3$
	+2.12	—600.7 div.
	+2.23	—580.2
	+3.22	—586.9
	+4.70	—579.6
	+3.60	—576.2
Means	+3.17	—584.7

Reduced to 0° with coeff. 1.83 div., we have

$$R_2 - Q_3 \text{ at } 0^\circ = -579.2 = -301.3\,\mu$$

Hence

$$Q_3 - 301.3\,\mu = R_2 = A_0 - 284.6\,\mu$$
$$Q_3 - \ 16.7\,\mu = A_0$$

COMPARISON OF CATHETOMETER METER Q_4 WITH R_2 AND R_3.

Equations of Condition between Q_4 and R_2.

No. Obs.	$R_2 - Q_4$		$R_2 - Q_4$ at 16°.67	Residuals.		
7	—644.1 div.	$= a + 16.19\,b$	—631.0 div.	—0.6 div.	=	—0.3 μ
7	—637.5	$= a + 12.69\,b$	—627.2	+3.2	=	+1.6
7	—638.4	$= a + 10.68\,b$	—629.7	+0.7	=	+0.4
3	—639.4	$= a + 6.24\,b$	—634.3	—3.9	=	—2.0
8	—618.9	$= a - 13.24\,b$	—629.6	+0.8	=	+0.4

Normal Equations.

$$-\ 3178.3 \text{ div.} = 5\,a + 32.56\,b$$
$$-21131.7 \text{ div.} = 32.56\,a + 751.46\,b$$

Hence

$$a = -630.4 \quad = 317.1 \ \mu$$
$$b = - \quad 0.81 = \quad 0.41 \mu$$

$$Q_4 - 317.1 \mu = R_2 = A_0 + 1.6 \mu$$
$$Q_4 - 318.7 \mu = A_0$$

$$\text{Coefficient} = +17.17 \mu - 0.41 \mu = 16.76 \mu$$

EQUATIONS OF CONDITION BETWEEN Q_4 AND R_3.

No. Obs.	$R_3 - Q_4$		$R_3 - Q_4$ at 16°.67	Residuals.		
7	-423.2 div.	$= a + 16.19 b$	-625.6 div.	$+2.8$ div.	=	$+1.4 \mu$
4	-472.2	$= a + 12.63 b$	-630.1	-1.7	=	-0.9
3	-552.1	$= a + 6.24 b$	-630.1	-1.7	=	-0.9
4	-800.2	$= a - 13.81 b$	-627.6	$+0.8$	=	$+0.4$

Normal Equations.

$$-2247.7 = \quad 4a + 21.25 b$$
$$-5209.8 = 21.25 a + 651.30 b$$

Hence

$$a = -628.4 \quad = -316.1 \ \mu$$
$$b = + \ 12.50 = + \quad 6.29 \mu$$

$$Q_4 - 316.1 \mu = R_3 = A_0 + 1.2 \mu$$
$$Q_4 - 317.3 \mu = A_0$$

$$\text{Coefficient} = +10.28 \mu + 6.29 \mu = +16.57 \mu$$

For the mean value we have:

$$Q_4 - 318.0 \mu = A_0$$
$$\text{Coefficient} = 16.66 \mu$$

For the relations between Q_1, Q_2, Q_4, and A_0, at 0° C., we have, from the comparisons made near 0°, as follows:

8612 Reading at Observation.	$R_2 - Q_1$ at 0°	$R_3 - Q_1$ at 0°	8612 Reading at Observation.	$R_2 - Q_2$ at 0°	$R_3 - Q_2$ at 0°
°			°		
$+0.65$	-291.3μ	-178.3μ	$+0.64$	-266.2μ	-158.0μ
$+4.13$	-292.3	-182.2	$+3.71$	-263.7	-151.7
$+10.08$	-293.1	-178.9	$+6.27$	-283.0	-154.6
Means	-292.2	-179.8		-264.3	-154.8

$$Q_1 - 292.2 \mu = R_2 \qquad\qquad Q_1 - 179.8 \mu = R_3$$
$$= A_0 - 284.6 \mu \qquad\qquad = A_0 - 170.2 \mu$$
$$Q_1 - \quad 7.6 \mu = A_0 \qquad\qquad Q_1 - \quad 9.6 \mu = A_0$$

For the mean $Q_1 - 8\ 6 \mu = A_0$

$$Q_2 - 264.3\,\mu = R_2 \qquad\qquad Q_2 - 154.8\,\mu = R_3$$
$$= A_0 - 284.6\,\mu \qquad\qquad = A_0 - 170.2\,\mu$$
$$Q_2 + 20.3\,\mu = A_0 \qquad\qquad Q_2 + 15.4\,\mu = A_0$$

For the mean $Q_2 + 17.8\,\mu = A_0$

8612 Reading at Observation.	$R_2 - Q_4$ at 0°	8612 Reading at Observation.	$R_3 - Q_4$ at 0°
°		°	
+ 0.48	$-324.2\,\mu$	+ 0.48	$-209.8\,\mu$
+ 3.98	-322.6	+ 3.98	-212.2
+ 5.94	-324.0	+10.43	-211.4
+10.63	-326.7		
Means	-324.4		-211.1

$$Q_4 - 324.4\,\mu = R_2 \qquad\qquad Q_4 - 211.1\,\mu = R_3$$
$$= A_0 - 284.6\,\mu \qquad\qquad = A_0 - 170.2\,\mu$$
$$Q_4 - 39.8\,\mu = A_0 \qquad\qquad Q_4 - 40.9\,\mu = A_0$$

For the mean $Q_4 - 40.3\,\mu = A_0$

Collecting results, we have:

At 0°	At 16°.67	Coefficient.
$W + 36.1\,\mu = A_0$	$W - 264.7\,\mu = A_0$	$18.19\,\mu$
$Q_1 - 8.6 = A_0$	$Q_1 - 306.4 = A_0$	17.87
$Q_2 + 17.8 = A_0$	$Q_2 - 289.2 = A_0$	18.45
$Q_3 - 16.7 = A_0$	. . . . .	. . .
$Q_4 - 40.3 = A_0$	$Q_4 - 318.0 = A_0$	16.71

It will be seen that every value of the coefficient is less than the value communicated by the Society, viz. 19.155 μ. It seems impossible to resist the evidence given by this determination, that this value is certainly too large. It is possible that the solution of the discrepancy may be found in the fact shown by these observations, that when the graduations are upon silver inlaid in brass, the coefficient of the brass controls that of the silver somewhat in proportion to the relative masses of the two metals. In every one of the five cathetometers standardized by the writer, the brass has carried the silver with it, that is, the coefficient has been found to be that of brass, and not that of silver.

It will be seen that the cathetometer has a less coefficient than the bronze bar R_2, which is identical in composition with the Imperial Yard.

It is probable, also, that the resultant coefficient of two metals which have a mechanical junction depends somewhat upon the perfection with which the junction is made. The writer has not found so large differences as here given in the values of the coefficient for different specimens of the same metal, but in this case the differences may be due to accidental errors of observation, on account of the limited number of comparisons made.

It should be stated also, that, judging by the color of the metal, the composition of W and Q_1 differs from that of Q_2 and Q_4.

It now only remains to describe some observations which were made to determine the extent to which the variations in the length of the standards under variations of temperature correspond with the computed values from the indicated readings of the thermometer when placed upon the upper surface of the bars.

First, a series of comparisons were made with a rising temperature. When reduced to 16°.67, the value of $R_2 - W$ should be 522.9 div. The following are the observed values.

	Time. h m.	8612 °	$R_2 - W$ at 16°.67	Deviation from 522.9 div.	
A. M.	7 10	6.20	—516.5 div.	— 6.4 div.	= —3.2 μ
"	8 20	7.30	—517.7	— 5.2	= —2.6
"	9 30	7.92	—522.1	— 0.8	= —0.4
"	11 20	12.07	—517.1	— 5.8	= —2.9
"	12 50	13.67	—511.3	—11.6	= —5.8

It therefore appears that, for a rising temperature of about 1° for each hour of time, the length derived from the reading of the thermometer will be from 2 μ to 3 μ too short.

The length of W was then compared with an end-meter in melting ice with a falling temperature. The temperature of the comparing-room had for several hours remained at 15°.4 C. At 8h. 27m. A. M. both windows were opened. The relation $S - W$ had been previously found to be —618.3 div. The comparisons were now continued every three minutes with the following results.

Time. A. M. h. m.	8612 °	$S - W$ at 16°.67 div.	Deviation from —618.3 div. div.	 μ
8 30	14.55	—634.2	+15.9	= + 8.0
8 33	14.25	—633.0	+14.7	= + 7.4
8 36	13.67	—645.6	+27.3	= +13.7
8 39	13.41	—639.8	+21.5	= +10.8
8 42	13.07	—639.3	+21.0	= +10.6
8 45	12.76	—646.1	+27.8	= +14.1
8 48	12.57	—639.9	+21.6	= +10.9
8 51	12.19	—643.1	+24.8	= +12.5
8 54	11.97	—631.0	+12.7	= + 6.4
8 57	11.39	—644.7	+26.4	= +13.3
9 0	11.10	—640.2	+21.9	= +11.0
9 3	10.94	—636.0	+17.7	= + 8.9
9 6	10.95	—623.3	+ 5.0	= + 2.5
9 9	10.90	—623.5.	+ 5.2	= + 2.6
9 12	11.00	—615.9	— 2.4	= — 1.2

It appears, therefore, that the measured length will be found to be too great under a rapidly falling temperature by about 10 μ or 12 μ for at least half an hour after the change of temperature has occurred.

Under circumstances similar to those described above, it will not be safe to make the comparisons under about one hour after the change of temperature. For a bronze bar having a cross section of one inch, the writer has found that this limit of time is about five hours. This form of the metres of the Society, therefore, seems to be well adapted for ordinary use. But it should be stated that the small depth of the bars *W* and Q_1 requires that they shall rest upon a flat surface during comparison, — something not easy to obtain.

In the third test, the temperature of bar *W* was raised to about 28° C. by heating over a register. At 3h. 19m. P. M. it was removed to the comparing room within which the temperature remained very constant at about 19°.0 C. Comparisons were then made every three or four minutes with the end-meter in melting ice, as follows. The comparisons were begun when the thermometer placed upon the bar ceased to rise.

Time. P. M. h. m.	8612 °	$S - W$ at 16°.67	Deviation from —618.3 div.		Time. P. M. h. m.	8612 °	$S - W$ at 16°.67	Deviation from —618.3 div.	
3 21	12.45	—620 div.	+17 div.	+ 1 μ	3 42	6.40	—647 div.	+29 div.	+15 μ
3 25	8.31	—676	+58	+29	3 44	6.21	—649	+31	+16
3 29	7.32	—676	+58	+29	3 46	6.09	—636	+18	+ 9
3 33	6.85	—667	+49	+25	3 49	6.00	—638	+20	+10
3 36	6.55	—660	+42	+21	3 51	5.77	—638	+20	+10
3 39	6.46	—658	+40	+20	3 54	5.49	—642	+24	+12

It will be noticed from these results, that the bar remained nearly two minutes in the comparing-room before the change of length was decidedly apparent. After that, the computed length was too great by a maximum amount of 29 μ, and notwithstanding the fact that after about fifteen minutes the thermometer readings decreased very slowly and with considerable regularity, the indicated length of the meter was at the expiration of 33 minutes still 12 μ too great. It is apparent, therefore, that, under circumstances similar to those described above, it will not be safe to make comparisons until the bar shall have remained at a nearly constant temperature for at least one hour.

The lines traced upon the silver surface are, in all these standards, of the best quality. The edges are not rounded, and it is possible to focus upon the lines with great sharpness. The writer has never before seen heavy lines of as good quality as these. The width of the lines in *W*, Q_1, and Q_2 is about 20 μ, and in Q_3 about 50 μ. In Q_4 the width is slightly different at the two ends, the mean value being about 25 μ.

The only serious criticism to be made upon these standards is, that the surfaces of W and Q_1 are slightly convex between centimeters 15 and 45, while Q_3 is convex to a less degree between centimeters 70 and 85. The surface of Q_4 is nearly plane when the cathetometer is supported at its neutral points.

RELATIVE ERRORS OF THE SUBDIVISIONS.

A plus sign indicates that the measured space is too short.

Decimeters.

Spaces.	W	Σ	Q_1	Σ	Q_2	Σ	Q_4	Σ
1	+ 0.7μ	+ 0.7μ	— 5.8μ	— 5.8μ	+ 4.7μ	+ 4.7μ	+ 0.0μ	+ 0.0μ
2	+11.3	+12.0	+ 1.9	— 3.9	+ 0.2	+ 4.9	— 2.5	— 2.5
3	+ 6.0	+18.0	—23.0	—26.9	—18.4	—13.5	— 1.1	— 3.6
4	—11.0	+ 7.0	+ 4.0	—22.9	— 0.1	—14.6	— 1.7	— 5.3
5	— 1.6	+ 5.4	+17.7	— 5.2	+ 0.9	—13.7	+ 9.1	+ 3.8
6	+ 3.0	+ 8.4	+ 5.3	+ 0.1	+ 0.2	—13.5	+ 1.4	+ 5.2
7	— 8.7	— 0.3	—11.1	—11.0	— 0.9	—14.4	— 9.3	— 4.1
8	+ 4.6	+ 4.3	—17.7	—28.7	+ 8.5	— 5.9	+ 8.6	+ 4.5
9	+ 1.3	+ 5.6	+11.4	—17.3	+ 4.3	— 1.6	+ 6.3	+10.8
10	— 5.3	+ 0.0	+17.3	+ 0.0	+ 1.6	+ 0.0	—10.8	+ 0.0

The relative errors of the 10 centimeters of the first decimeters of Q_3 and Q_4 were found to be as follows. The centimeter subdivisions of W and Q_1 were not investigated.

Spaces.	Q_3	Σ	Q_4	Σ
1	+0.5μ	+0.5μ	+1.4μ	+1.4μ
2	—5.0	—4.5	—1.1	—0.3
3	—2.8	—7.3	—4.7	—5.0
4	—0.8	—8.1	—4.0	—9.0
5	+1.9	—6.2	+4.1	—4.9
6	+0.3	—5.9	+5.1	+0.2
7	+1.1	—4.8	—3.1	—2.9
8	+3.1	—1.7	+0.4	—2.5
9	+1.3	—0.4	+4.9	+2.4
10	+0.4	+0.0	—2.4	+0.0

The errors of the one-tenth millimeter subdivisions seem to be inappreciable, indicating that the screw employed has no sensible periodic error which is a function of a single revolution.

With regard to the decimeter subdivisions, it should be said that the graduation was done in three operations, the second zero being at about the thirty-third centimeter, and the third at about the sixty-sixth centimeter.